

## Problem Set 9

### Teams

#### Main Points

- **Free Rider Problem:** The main problem with team-based compensation is the free-rider problem which arises because the agents don't consider the impact of their actions on the total outcome but only their own payoff.
- **Peer Pressure:** Peer pressure can act as a powerful incentive tool, by affecting either internal ('guilt') or external ('shame') motivation that can be used by teams to address the free rider problem.
- **Difference-in-Difference (DD) Method:** The DD method consists of comparing the change in outcomes for two groups, of which one is affected by treatment and the other is not, before and after the treatment is introduced. The DD estimate will identify the treatment effect provided the common trend assumption is satisfied (i.e. the change in the outcome for the two groups would be the same if there was no treatment).

#### Main Concepts

Free rider problem; Peer pressure; Social/Group norms; Difference-in-Differences; Common trend assumption.

#### Problems

- (1) Two accountants are about to form a partnership. The revenues of the partnership  $Q$  depend on the effort of each accountant according to  $Q=e_1+e_2+u$ , where  $u$  is a random variable with a mean of zero. The cost of effort is identical for both accountants:  $c(e_i)=0.5e_i^2$ . The accountants decided that they will split the revenues equally. What is the loss in the expected social surplus due to the free rider problem?
- (2) Consider a partnership of  $N$  agents, each with the cost of effort function  $0.5e_i^2$ . Each agent contributes  $q_i=e_i$  to the total output  $Q= e_1+e_2+ \dots +e_n+ u$ , where  $u$  is a random variable with a mean of zero. Show that the loss in the social surplus due to the free rider problem increases in the size of partnership  $N$ .
- (3) Consider the partnership described in Problem 1. Suppose now that the two accountants can impose a penalty on each other, given by  $\gamma(G-e_i)^2$ , where  $\gamma$  denotes guilt (or shame) and  $G$  is the group norm. Explain how would the partners set the group norm and how does the norm depend on  $\gamma$ .
- (4) Employee's output in Coca Cola is given by  $q=e+u$ , where  $e$  is effort and  $u$  is a random variable with a mean of zero. Each Coca Cola employee is paid individually according to  $w=q$ . Employees in Pepsi work in teams of 2. Each employee's output is  $q=e+n+u$ , where  $n$  is employee's ability. Pepsi employees get equal share of their total output  $Q=q_1+q_2$ . The cost of effort function is  $0.5e^2$  for employees in both Coca Cola and Pepsi. All employees are risk neutral and don't exert peer pressure on each other. All employees in

the economy are of two ability types: n=0 or n=0.2. What is the predicted observed difference in productivity per employee between Coca Cola and Pepsi?

- (5) Prior to 2010, the insurance companies in Ontario paid its sales associates based on the number of insurance contracts that each associate sold. After 2011, these companies switched to a team based compensation. In contrast, the insurance companies in Quebec paid its sales associates based on the number of insurance contracts both before and after 2010. To estimate the impact of this change, a researcher has collected data from insurance companies in Ontario and Quebec and estimated the following model:

$$\text{Productivity} = a + b \times \text{Ontario} + c \times \text{Post2010} + d \times \text{Ontario} \times \text{Post2010}$$

where Ontario is 1 if the firm is in Ontario and 0 if it is in Quebec, and Post2010 is 1 for the period after 2010 and 0 otherwise. The results were as follows:

Variable	Coefficient	Standard Error
Constant	10	2
Ontario	1	1
Post2010	3	1
Ontario×Post2010	2	0.5

- a. Define the hypothesis to be tested and how it relates to theory.
- b. Define treatment and the treatment and control groups.
- c. Identify assumptions needed to strengthen inference about the causal impact of treatment.
- d. Interpret the results (e.g. the statistical and economic significance of estimates; the relation between the results and specific theoretical predictions).

**Suggested Solutions**

(These solutions are intended to be accurate and as complete as possible. Please report any remaining errors to [jasmin.kantarevic@oma.org](mailto:jasmin.kantarevic@oma.org) ).

(1) The efficient level of effort for each accountant satisfies the first-order condition  $E[q'(e_i^*)]=c'(e_i^*)$ , or  $1=e^*$ . Therefore, the expected social surplus is  $E[Q]-c(e_1)-c(e_2)=1+1-0.5(1^2)-0.5(1^2)=1$ . If the partners share the revenues equally, the privately optimal level of effort for each partner satisfies the first-order condition  $0.5E[q'(e_i^*)]=c'(e_i^*)$ , or  $0.5=e^*$ . Therefore, the social surplus becomes  $E[Q]-c(e_1)-c(e_2)=0.5+0.5-0.5(0.5^2)-0.5(0.5^2)=3/4$ . Therefore, the expected loss in the social surplus is  $1-3/4=1/4$ .

(2) The efficient level of effort for each agent satisfies the first-order condition  $E[q'(e_i^*)]=c'(e_i^*)$ , or  $1=e^*$ . Therefore, the expected social surplus is  $E[Q]-\sum_i c(e_i)=\sum_i (e_i-0.5e_i^2)=N(e-0.5e^2)=N[1-0.5(1^2)]=N/2$ . If the agents share the revenues equally, the privately optimal level of effort for each agent satisfies the first-order condition  $(1/N)E[q'(e_i^*)]=c'(e_i^*)$ , or  $1/N=e^*$ . The social surplus is then  $E[Q]-\sum_i c(e_i)=\sum_i (e_i-0.5e_i^2)=N(e-0.5e^2)=N[(1/N)-0.5((1/N)^2)]=1-1/2N$ . Therefore, the expected loss in the social surplus is  $N/2-1+1/2N=[(1+N^2)/2N]-1$ . The derivative of this expression with respect to  $N$  is  $(2N^2+1)/(2N)^2$ , which is positive. Therefore, the expected loss in the social surplus due to the free rider problem increases with the size of the team  $N$ .

(3) Each accountant  $i$  now solves  $\text{Max } 0.5(e_i + e_j) - 0.5e_i^2 - \gamma(G-e_i)^2$ . The first-order condition for  $e_i$  is  $0.5-e_i+2\gamma(G-e_i)=0$ . The accountants will choose the group norm  $G$  to induce the efficient level of  $e$ ,  $e^*=1$ . Solving the first-order condition for  $G$  then yields  $G=1+(1/4\gamma)$ . The norm depends negatively on  $\gamma$ . Specifically, the more shame or guilt the partners feel about deviating from the norm, the smaller the norm is needed to achieve the efficient level of effort.

(4) Employees in Coca Cola of any ability level will choose  $e^*=1$ . Their expected payoff is then  $E[w-c(e)]=1-0.5(1^2)=0.5$ . An employee of any ability  $n$  in Pepsi will choose  $e=0.5$ . The expected total output is then  $Q=0.5+n_1+0.5+n_2=1+n_1+n_2$ . Specifically, if both workers are of ability  $n=0.2$ , the output is 1.4; if both workers are of ability  $n=0$ , the output is 1; and if the workers have different abilities, the output is 1.2. Now, if the worker is of ability  $n=0.2$  and pairs with a worker of same ability, her expected payoff is  $0.5(1.4)-0.5(0.5^2)=0.575$ . If the worker is of ability  $n=0.2$  and pairs with a worker of ability  $n=0$ , her expected payoff is  $0.5(1.2)-0.5(0.5^2)=0.475$ . Similarly, if a worker of ability  $n=0$  pairs with a worker of ability  $n=0.2$ , his expected payoff is  $0.5(1.2)-0.5(0.5^2)=0.475$  and if he pairs with a worker of ability  $n=0$ , his expected payoff is  $0.5(1)-0.5(0.5^2)=0.375$ . Therefore, it is better for employees of ability  $n=0.2$  to work for Pepsi and pair in teams with workers of same ability and it is better for employees of ability  $n=0$  to work for Coca Cola. The average productivity of workers in Coca Cola would then be equal to  $E[q]=e=1$ . The average productivity of workers in Pepsi would instead be equal to  $E[q|n=1]=e+n=0.5+0.2=0.7$ .

(5) (a) The hypothesis to be tested is whether the workers' productivity in firms with the team-based compensation is different than in firms with the individual, piece rate compensation. From the theory of incentives that we study in the course, we know that the productivity of workers in the piece rate compensation depends on the extent to which the pay is tied to performance, which in turns depends on a number of factors such as risk preferences, the extent of uncertainty, the accuracy of performance measures, availability of multiple signals, multitasking, and presence of non-financial incentives. On the other hand, the productivity of workers in the team-based compensation firms depends on the strength of free-rider problem and the extent to which

workers can and do exert peer pressure. In general, it is not clear whether the productivity should be higher or lower in one type of firms as compared to the other (i.e. it remains an empirical question). **(b)** The treatment in this problem could be defined as the team-based compensation. The treatment group is the set of firms in Ontario, while the control group is the set of firms in Quebec. The reason for this classification is because firms in Ontario were treated (i.e. they switched from the individual to team-based compensation, while firms in Quebec were not treated (i.e. they remained on the individual based compensation throughout the sample period)). **(c)** The empirical method in this problem is the difference-in-differences methodology. The main assumption in this methodology is known as the common trend assumption: firms in Ontario and Quebec would have experienced similar change in productivity over time if there was no treatment (i.e. firms in both provinces did not change their compensation model). **(d)** The main coefficient of interest is the interaction between Ontario and Post2010 since it identifies the treatment group in the post-treatment period. The estimate is 2, which is statistically significant ( $t\text{-statistic}=2/0.5>2$ ), indicating that workers' productivity is higher when they are paid based on group rather than individual output. In addition, the productivity of workers in Ontario prior to the introduction of the team-based compensation was equal to  $10+1=11$ , so the percentage increase in productivity is about  $2/11$  or about 18%. This seems to be economically significant.